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THE THEORY OF THE SCREW PROPELLER.*

By

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The reason why the comprehension of what occurs in the vicinity of a propeller is commonly regarded as especially difficult, does not lie so much in the complexity of the hydrodynamical phenomena as in our limited ability of geometrical presentation, which sometimes fails us, even in simple cases, when these have a spiral form. The mathematical treatment is rendered still more difficult by problems connected with the propeller symmetry. Aside from the above inconveniences, which are not inherent in the nature of the phenomena, the propeller offers no greater difficulties than the majority of other hydrodynamic problems. By confining ourselves to the most essential phenomena, we can represent them in very simple fashion. It is characteristic of this simplicity that we have long had a presentation of the theory of the propeller in Rankine's propeller slip-stream theory, which is a fairly close approximation to the truth and which may be regarded as a sort of forerunner of the modern theory** of aerofoils. Only when we endeavor to acquire a more accurate knowledge

* Reprint from "Die Naturwissenschaften," 1921, No. 18.

** Rankine, "On the Mechanical Principles of the Action of Propellers." Transactions of the Institution of Naval Architects, 1865, Vol. VI, p.13. The theory was considerably improved later, especially by Froude. Froude, "On the Part Played in Propulsion by Differences of Fluid Pressure." Transactions of the Institution of Naval Architects, 1889, Vol. XXX, p.390.

of the phenomena, will the theoretical treatment become more difficult. Our knowledge is most deficient in regard to the mutual action of propeller and airplane. In this connection, we must depend almost wholly on experiments. The cause of this ignorance resides less, however, in the propeller, than in the resistance of the airplane, which here plays a very important role. This problem of the resistance of a body moving in a fluid and the attendant loss of energy, aside from any special cases, has thus far withstood all theoretical treatment and has had to be worked out almost exclusively by means of experiments. It is therefore natural that our imperfect knowledge of the phenomena which determine the resistance of the aircraft, also hinders the theoretical treatment of the related propeller phenomena. The problems relating to the independent operation of the propeller are, on the contrary, with the exception of a few very special problems, mostly solved, at least as regards propellers with favorable shapes, which alone are of any practical importance. This limitation has the advantage of enabling us to assume as small the loss of energy from friction and the formation of vortices, over which we have the least control, as compared with the amount of energy otherwise lost, and the further special advantage that the large number of shapes which otherwise would have to be considered is thereby greatly reduced. The following exposition will consist of a brief review first of the fundamental principles of the propeller slip-stream theory and its further development through later researches, which demonstrate the connection between the propeller slip-stream theory and Froude's so-called "propeller blade theory."*

If we wish to exert a force on a body, for example, on a vehicle to overcome the head resistance, we must use some other body as a brace and exert upon it the same force, but in the opposite direction (law of action and reaction). In moving a vehicle on the solid earth, the latter is nearly always the resisting body, which, on account of its large mass, suffers no noticeable change in speed from the reaction. The case is different, if we utilize a relatively small body which is not rigidly connected with the earth. In such a case, the body is affected by the force of reaction and acquires a noticeable velocity. This phenomenon is very apparent in firing a cannon ball. We wish to impart a velocity to the ball and therefore a force must be exerted upon it. The cannon is the resisting body and is given a velocity in the opposite direction to that of the cannon ball (recoil). The case is similar if we wish to set in motion a body in a fluid (air or water) by means of a propeller. We may think of the process as follows: A mass m of the fluid is utilized for a second as the reacting body and acquires a certain velocity v. Then another equal mass of fluid is brought into play and serves as the reacting body for the next second, etc., so that in each second a mass m acquires an increase in velocity v. If S represents the propeller thrust, the force of reaction must have the same

* Froude, "On the Elementary Relation between Pitch, Slip and Propulsive Efficiency." Transactions of the Institution of Naval Architects, 1878, Vol. XIX, p.47.

value and the acceleration v, imparted to the mass m in one second by the action of this force, is $v = S/m$.

It is not essential for us to think of the process as being divided into intervals of one second. If we represent each interval by $1/n$th of a second and take the corresponding $1/n$th part of the mass, we obtain for the same reaction force, the same velocity and the mass accelerated <u>per second</u> remains the same. The velocity imparted to the reacting fluid is directly proportional to the thrust and inversely proportional to the mass of fluid acted upon per second. With this velocity, the fluid acquires an increase in energy amounting to $1/2\, m\, v^2$ per second. This energy must be supplied in addition to the useful power, in order to obtain the desired thrust. It means, however, an unavoidable loss of energy.

If all other losses are disregarded, it follows from an extension of the above reasoning that the most favorable case is when the thrust is evenly distributed over the whole surface of the propeller blades. This condition would be approximately fulfilled by a propeller with very many blades. The theorems for the losses in the propeller slip stream obtained from this very simple theory, are very useful for estimating the efficiency of a propeller, since the remaining losses are ordinarily considerably smaller and not so dependent on external conditions.

The theory also gives the velocity with which the fluid passes through the plane of the propeller. It may be shown that the fluid passing through the propeller acquires half its acceleration in front of the propeller and the other half behind it. An accurate knowledge of the flow at the plane of the propeller is however much desired, since it will give us a basis for calculating the blades and the position to be given them in order to obtain the desired thrust. For this purpose, however, the results of this primitive theory are no longer entirely adequate. So long as but little was known concerning the action of the fluid on the propeller blade, there was no great need of a more accurate knowledge of the flow in the plane of the propeller. But after the investigation of the phenomena of aerofoils had laid in this respect the foundation for a more accurate calculation of the propeller blade, it was also desirable to increase the knowledge of the flow in the vicinity of the propeller.

There were in the main two points requiring further elucidation. In the first place, the propeller, in addition to the motion of the slip stream parallel to its axis associated with the thrust, also generates tangential motions which necessitate a slight correction to the considerations of energy and, what is more important, produce a noticeable increase in the flow through the propeller disk. In the second place, screw propellers always have had a very limited number of blades. It was therefore desirable to determine what difference this circumstance makes in comparison with the assumption of a large number of uniformly distributed narrow blades. In both directions considerable progress has recently been made. Although some points have not yet been worked out for convenient practical application, the principal difficulties have nevertheless been overcome.

The investigation of the rotation of the propeller slip stream is closely connected with the above-mentioned considerations of the simple older theory. Simply, the propeller torque is substituted for the thrust. The connection between the individual quantitiss is indeed considerably more complex in this extended propeller slip-stream theory, than in the older theory, and the calculations are more difficult. But after the requisite laborious calculations have once been made, the results can be expressed in the form of curves, which can serve as the basis for practical applications.*

The second point, in which the old propeller slip-stream theory needed to be supplemented, was the assumption that the thrust could be distributed at will over the surface of the propeller disk, which holds true to a certain degree for a propeller with very many narrow blades, but certainly not for an airplane propeller with two blades which cover only a very small portion of the propeller disk. It may however be here noted that the difference in comparison with the uniform distribution is not so great as appears at the first glance. On account of the revolution of the propeller, its blades exert a pressure at every point of the propeller disk, only not simultaneously and continuously, but periodically always again at another place.

For the treatment of this propeller with widely separated

*Betz, Eine Erweiterung der Schraubenstrahltheorie, Zeitschrift für Flugtechnik und Motorluftschiffahrt, 1920, Vol. CI, p.105.

blades, there is a very useful method which was developed principally in connection with the theory of aerofoils and has already been very successfully applied in that connection (Compare the article, Betz, "Einführung in die Theorie der Flugzeug-Tragflügel," Die Naturwissenschaften, Vol. 6, p.557). A field of well-defined vortices is connected with the distribution of the propeller thrust, or the lift of a wing. Since, on the other hand, the motion of the fluid is definitely determined by the vortices existing in it, we can calculate the flow from the thrust distribution by means of this concept of vortices.

Such a calculation, however, consumes considerable time. An effort has been made, therefore, to simplify this work. Föttinger gave a practically applicable method in his lecture before the Society of Naval Engineers (Schiffbautechnische Gesellschaft) in 1917.* He proceeded from the correct concept that the strongest vortices are restricted to definite regions, so that we can represent them approximately by single vortex lines. These lines are the propeller axis and the spiral lines going out from the tips of the propeller blades and encircling the propeller axis (Fig. 1).

As in the theory of aerofoils, in which a corresponding approximation is employed, this simple vortex picture always performs very good service, when the flow is investigated at some distance from the vortex lines. That the picture of the flow in the vicinity of the individual vortex lines can no longer agree

* Föttinger, Neue Grundlagen für die theoretische und experimentelle Behandlung des Propellerproblems, Jahrbuch der Schiffbautechnischen Gesellschaft, 1918, Vol. 19, p.385.

with reality, follows from the fact that the theoretical velocity in the immediate neighborhood of an infinitely thin vortex line is infinitely large. If we wish to investigate the flow in the vicinity of the vortex field, more especially, for example, at the place where the blade itself is, we must have as a basis more accurate data on the distribution of the vortices, that is, on the distribution of the thrust along the blade.

After it had been demonstrated by the theory of aerofoils (in which the same difficulties occur, though to a lesser degree) that the most favorable lift distribution gave very simple flow relations, the idea suggested itself to investigate as to whether, for the propeller also, the most favorable thrust distribution is not characterized by very simple flow relations. As a matter of fact, perfectly analogous laws for the propeller can be derived by proper modifications of those employed for aerofoils.*

The most important one of these laws reads: The flow behind a propeller which has the least loss of energy agrees with the ideal flow about rigid screw surfaces displaced axially backwards. The shape of these screw surfaces is that which is cut in the fluid by the propeller blades in their motion. The speed of the displacement depends on the magnitude of the thrust.

However simple this statement concerning the flow generated by a propeller with the most favorable thrust distribution may appear, the problem is nevertheless not entirely solved by it. The

* A. Betz, Schraubenpropeller mit geringstem Energieverlust, with an appendix by L. Prandtl, Nachrichten der Gesellschaft der Wissenschaften zu Gottingen, Math. physik. Kl. 1919, p. 193.

mathematical treatment of the flow about such a displaced screw surface presents very great difficulties. Prandtl has given an approximate solution of this problem in an appendix to the writer's article mentioned in the footnote. Though this solution does not give entirely accurate values, especially for the two-bladed propeller, it is perfectly satisfactory however for practical purposes.

Fig. 2 represents the most favorable thrust distribution, as determined by the old propeller slip-stream theory and also by the later improvements of the same. The thrust per unit surface is represented for the different distances r from the axis of the propeller. In case c, the thrust, which is here concentrated on the blade, is to be thought of as uniformly distributed on the circumferences belonging to the corresponding radii. In the old propeller slip-stream theory, the thrust is evenly distributed over the whole surface. With the consideration of the slip-stream rotation, we obtain a pressure drop at the axis and, with the consideration of the finite number of blades, we obtain also a pressure drop at the blade tips.

In deducing the laws of the screw propeller with the most favorable thrust distribution in the case of a finite number of blades, it has been assumed that the thrust is so small, that the flow velocities generated by the propeller can be regarded as small in comparison with the proper motion of the propeller. We can, however, by foregoing strict mathematical accuracy, but without any error worth mentioning, so modify the laws that they will also hold good for more heavily loaded propellers.

In all the preceding discussion it has been taken for granted that we possess in the propeller blades a suitable device for exerting forces on the air, which will produce the desired thrust. No knowledge has been gained however as to the necessary shape of the blades for obtaining the desired effect nor as to the loss of energy due to the production of pressure by the blades. These data may be supplied by a method entirely different from the preceding. Its principles were expounded by Froude in 1877. Both theories, the propeller slip-stream theory and the propeller-blade theory, long existed side by side, without our being able to give an entirely satisfactory explanation of the real connection between them. Knowledge of the phenomena in the vicinity of aerofoils, and more especially of the influence of the span on the resistance, first shed light on the corresponding phenomena in the vicinity of propellers.

If we assume that the effects of the individual parts of the blade, in its motion through the field, are independent of each other, we only need to determine experimentally, once for all, the forces arising in connection with the motion of definite cross sections, in order to calculate from them the forces acting at every point of the blade. The magnitude and direction of the velocity of the particular portion of the blade is indeed given by the two components, the circumferential and the forward motion. From the forces acting on the individual portions of the blade, the thrust and torque of the whole propeller can then be readily calculated. In practice, this method has the important advantage of giving the relation between the thrust and the torque, on the one hand, and the shape of the blade, on the other. Unfortunately, the assumption that the individual parts of the blade do not interfere with each other in their effect, is not correct. In practice, we may, to a certain extent, avoid this difficulty by taking as the basis of the calculation for each propeller type, somewhat different section characteristics, so selected that the resulting values for the whole propeller agree with the experimental values.

The propeller slip-stream theory, especially in its improved form, now gives us the basis for determining the mutual influence of the parts of the blade, so that, in calculating the shape of the blade, we can get along with certain section characteristics, which have been determined once for all. As we have already seen, the fluid has acquired a certain added velocity in passing through the propeller disk. Consequently the motion of the blade section relative to the fluid is dependent not only on the forward and circumferential speed, but also on this proper velocity of the fluid. In this acquired velocity expression is however found for the whole influence exerted by the remaining parts of the propeller on the effect of an individual section. The connection between the acquired velocity, and the section characteristics is especially manifest, if we compare the loss of energy, as given by the propeller slip-stream theory, with the loss appearing at the blade in consequence of this added velocity. When we thus combine the propeller slip-stream and propeller-blade theories, we obtain a complete theory, corresponding well with facts, of the screw propeller operated by itself. The propeller slip-stream theory gives the action of an ideal propeller which, among other things, forms the basis for the correct application of the propeller-blade theory. The latter clears up the additional phenomena which depend on the special properties of the blades and more especially the magnitude of the losses at the propeller blade, not considered in the propeller slip-stream theory.

It is intended to show, by the above explanations, that the new theories present the possibility of investigating the phenomena in the vicinity of a propeller, so as to be able to calculate its action on the basis of fewer experimental values. As already mentioned, there is still much work to be done in reducing the methods to a convenient form for practical application. There is also much still to be investigated experimentally. Aside from all questions concerning the mutual effect of propeller and airplane, there are the characteristics of the blade sections, which still require thorough investigation. Although we have very accurate values for airplane wings, it is still uncertain whether these values can in all cases be applied in their present form to propellers. The centrifugal force of the revolving propeller may well cause deviations. Furthermore, at high velocities, the compressibility of air, as likewise the so-called cavitation in water, plays a certain role. There is still, therefore, in spite of all theoretical progress, a rich field for experimental activity in connection with the screw propeller.

Translated by the National Advisory Committee for Aeronautics.

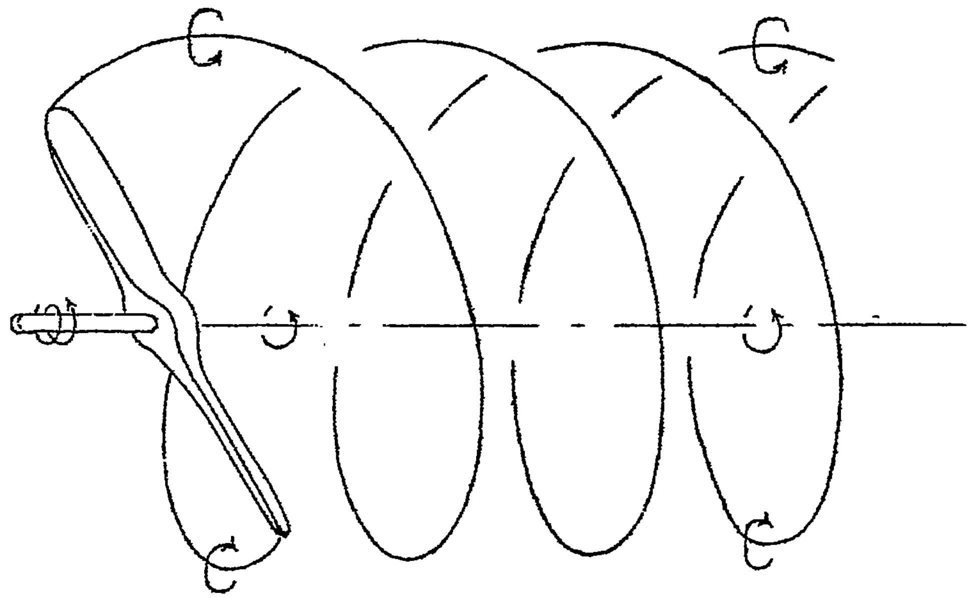

Fig. 1 - The system of the most important vortices behind a screw propeller.

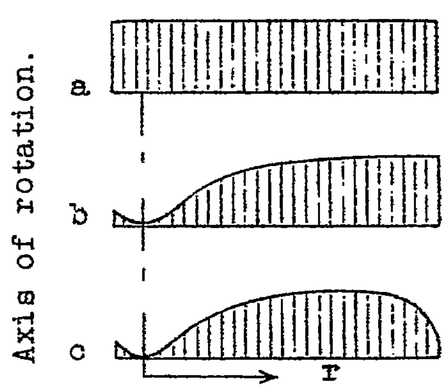

Fig. 2 - Most favorable thrust distribution over the propeller disk:
a) according to the simple propeller slip-stream theory;
b) considering rotation of slip-stream;
c) considering finite number of propeller blades.